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A SIMPLE AND ROBUST METHOD FOR CALCULATING RETURN PERIODS OF OCEAN WAVES

Edward B.L. Mackay Renewable Energy Research Group, University of Exeter Penryn, Cornwall, UK e.mackay@exeter.ac.uk Lars Johanning Renewable Energy Research Group, University of Exeter Penryn, Cornwall, UK <u>I.johanning@exeter.ac.uk</u>

ABSTRACT

A new method is introduced for combining the long-term distribution of sea states with the short-term distribution of individual wave or crest heights, conditional on sea state. The method uses a Monte Carlo approach to simulate random realisations of the maximum wave or crest height in each sea state. A peaks-over-threshold analysis is conducted on the random maxima in each sea state in order to estimate the longterm distribution of individual wave or crest heights. The new method is significantly simpler than existing methods such as the equivalent storm approach, requires fewer assumptions and has similar computational times. The new method is applied to a 35 year dataset of wave buoy measurements and is shown to produce almost identical estimates of return values of individual crest heights to the equivalent storm method.

1. INTRODUCTION

Calculating the long-term statistics of extreme individual wave and crest heights is an important problem in coastal and ocean engineering. To calculate the return periods of individual wave or crest heights, the long-term distribution of sea states must be combined with the short-term distribution of individual wave or crest heights conditional on sea state. The methods for combining the long-term and short-term distributions are identical for both wave and crest heights, so to avoid referring to both throughout the work, the following discussion will be presented in terms of wave heights rather than crest heights.

The various methods proposed for combining these distributions can be grouped into three categories, according to the 'counting' method used. In the first category, the distribution of all individual wave heights in a given time period is calculated [1, 2]. In the second category, the distribution of the maximum wave height in each sea state is calculated [3, 4(Section 6.4.2)]. In the third category the distribution of the maximum wave in each storm is calculated [5-12]. Forristall [13] compared various models for combining the long-term and short-term distributions

against long time series of simulated individual wave heights and concluded that storm-based models give the most accurate estimates of return periods of individual wave heights. The DNV GL guidelines [14] also recommend the use of equivalent storm methods for estimating extreme individual wave heights.

In a storm-based model the distribution of the maximum wave in each measured storm is calculated and a parametric representation of this distribution is fitted using an 'equivalent storm' model (see [12] for a review of equivalent storm models). After each measured storm has been fitted with an equivalent storm, the joint distribution of the equivalent storm parameters must be estimated. Return periods of storms in which a given wave height is exceeded are then calculated by combining the joint distribution of equivalent storm parameters with the distribution of the maximum height in the storm, conditional on the storm parameters.

There are several drawbacks to equivalent storm method. Firstly, the accuracy of the model is dependent on the accuracy of the fit of the equivalent storm models to the measured storms. Secondly, a model must be established for the joint distribution of the equivalent storm parameters. Finally, a double or triple integral may be required to combine the joint distribution of storm parameters with the distribution of the maximum wave height in the equivalent storm, which can be complicated to compute (although in some cases this double or triple integral can be reduced to a single integral).

In this paper, an alternative method is proposed for combining the long-term and short-term distributions, which gives the same results as the equivalent storm method, but without the disadvantages mentioned above. The method proposed here uses a Monte Carlo approach, where multiple random realisations of the maximum wave height in each sea state are generated and a peaks-over-threshold analysis is conducted on each random realisation. The method does not require calculating the distribution of the maximum wave height in measured storms, fitting equivalent storms, estimating joint distributions or combining distributions via integration. It is therefore much simpler to implement and more robust, since there is only one fitting stage rather than three in the equivalent storm method.

The paper is organised as follows. Section 2 presents the equivalent storm method, which is used as a baseline for comparison. The Monte Carlo method is presented in Section 3. An example application of the methods to measured wave data is presented in Section 4, and the two methods are compared. Finally, conclusions are presented in Section 5.

2. THE EQUIVALENT STORM METHOD

The first step in the equivalent storm (ES) method is to identify separate storms, in which the maximum wave height can be considered as effectively independent events. The criteria used to define separate storms typically state that the time between the peak significant wave height, H_s , of two adjacent storms must be larger than some minimum value and that the minimum H_s between two adjacent peaks must be less than some multiple of the lower of the peak. It is possible to derive criteria for defining independent storms in a more rigorous manner, using the extremogram [15], which is an analogue of the autocorrelation function for extreme events. For most locations, the extremogram shows that storm peak wave heights separated by around 5 days can be considered effectively independent. For the present work, storms are defined using a minimum temporal separation of 5 days between adjacent peaks.

Once the time series of sea states has been divided into separate storms, the distribution of the maximum wave height in each storm can be calculated. First it is necessary to introduce some notation. A storm will be defined as a sequence of discrete sea states, σ_i , with associated sea state parameters $H_s(i)$, $T_z(i)$, $T_m(i)$, etc. Sea state parameters are defined here in the usual way: significant wave height $H_s = 4\sqrt{m_0}$, zero up-crossing period $T_z = \sqrt{m_0/m_2}$, mean period $T_m = m_0/m_1$ and $m_n = \int_0^\infty f^n S(f) df$ is the n^{th} moment of the wave frequency spectrum, S(f). The cumulative distribution function (CDF) of individual wave heights, conditional on sea state is denoted $F_{\sigma}(h) = \Pr(H \le h | \sigma)$. For the present discussion, the form of the short-term distribution $F_{\sigma}(h)$ is not important. Examples using specific forms of $F_{\sigma}(h)$ will be presented in Section 4.

The distribution of the maximum wave height in each sea state is calculated by assuming that individual wave heights are independent. Under this assumption, the probability that the maximum wave height in a sea state, H_{max} , does not exceed a level *h* in an interval Δt , is simply the product of the probability that each individual wave does not exceed *h*. The number of waves, *N*, in the interval Δt can be estimated as $N = \Delta t/T_z$, where T_z is the mean zero up-crossing period.

$$\Pr(H_{max} \le h | \sigma_i) = F_{\sigma_i}(h)^N \tag{1}$$

The distribution of the maximum wave height in a storm is calculated by assuming independence of the maxima in each sea state. Under this assumption, the probability that the maximum wave height in a measured storm, MS, does not exceed level h, is calculated as the product of the probabilities that the maximum wave height is not exceeded in any of the sea states over the course of the storm:

$$\Pr(H_{max} \le h|MS) = \prod_{i=1}^{k} \Pr(H_{max} \le h|\sigma_i)$$
(2)

where the storm is defined as sea states $\sigma_1, ..., \sigma_k$.

The next step is to parameterise the distribution (2) in some way, with the parametric representation referred to as the equivalent storm. Two types of method have been proposed for modelling the distribution of the maximum wave height in the measured storm. One approach is to model the temporal evolution of sea states in a storm using some simplified geometric form, such as a triangle [6, 7], trapezoid [10], parabola [4, Section 6.54], power law [8, 9] or exponential [11]. The parameters of the equivalent storm are fitted so that the distribution of the maximum wave height in the equivalent storm is matched as closely as possible to the measured storm. This approach is reasonably effective because the order of sea states in the storm does not affect the distribution of the maximum wave height. Therefore, the product in (2) can be re-ordered into a monotonically increasing series of sea states, for which a linear, power or exponential law is a reasonable fit.

However, modelling the temporal evolution of sea states is not necessary and the distribution of the maximum wave height in the storm can be modelled directly. This approach was adopted by Tromans and Vanderschuren [5], who assumed that the square of the maximum wave height in the storm followed a Gumbel distribution, with a fixed relationship between the scale and location parameters. Mackay [12] developed this idea and used the generalised extreme value distribution (GEV) to model the distribution of the maximum wave height in a storm, without assuming any fixed relations between the GEV parameters *a priori*. In [12] it was demonstrated that using the GEV to model (2) improves the goodness-of-fit by an order of magnitude compared to both temporal evolution methods and T&V method. The GEV will therefore be used here as the equivalent storm model.

The CDF of the maximum wave height in the equivalent storm is defined in terms of the GEV as:

$$\Pr(H_{max} \le h|ES) = \begin{cases} \exp\left(-\left(1+k\left(\frac{h-a}{b}\right)\right)^{-\frac{1}{k}}\right) & \text{for } k \ne 0 \\ \exp\left(-\exp\left(-\left(\frac{h-a}{b}\right)\right)\right) & \text{for } k = 0 \end{cases}$$
(3)

where a, b and k are the location, scale and shape parameters, respectively. The GEV can be fitted to (2) by finding the parameters that minimise the Cramér–von Mises goodness-of-fit

parameter, ω , which quantifies the difference between two distributions:

$$\omega^2 = \int_0^\infty [\Pr(H_{max} \le h|MS) - \Pr(H_{max} \le h|ES)]^2 dh \qquad (4)$$

The fitting procedure has been implemented in MATLAB using the function '*fininsearch*', which uses a simplex search algorithm [16]. The starting point for the search is the moment estimators for the Gumbel distribution, i.e. taking k = 0 and calculating a and b as functions of the expected value and variance of H_{max} in the measured storm.

Once all measured storms have been parameterised, the next step is to model the joint distribution of the equivalent storm parameters. The probability density function (PDF) of the joint distribution of storm parameters, p(a, b, k), can be written as

$$p(a,b,k) = p(a)p(b,k|a)$$
(5)

where p(a) is the marginal PDF of a and p(b, k|a) is the joint PDF of b and k conditional on a. It will be shown in Section 4, that the shape parameter, k, is uncorrelated to a and b, and can be assumed independent, so the joint PDF can be further simplified as

$$p(a, b, k) = p(a)p(b|a)p(k)$$
(6)

The estimation of p(b|a) and p(k) will be discussed in Section 4. The distribution of the GEV location parameter, a, is established using a peaks-over-threshold (POT) analysis. In the POT method, the generalised Pareto distribution (GPD) is fitted to observations exceeding a high threshold. The CDF of the GPD is defined conditional on a exceeding some high threshold u:

$$\Pr(a \le x | a > u) = \begin{cases} 1 - \left(1 + \xi \frac{x - u}{\sigma}\right)^{-\frac{1}{\xi}} & \text{for } \xi \neq 0 \text{ and } \sigma > 0 \\ 1 - \exp\left(-\frac{x - u}{\sigma}\right) & \text{for } \xi = 0 \text{ and } \sigma > 0 \end{cases}$$
(7)

When $\xi \ge 0$ the support (that is, the range of values over which the distribution is defined) is $0 \le x < \infty$. When $\xi < 0$ the support is $0 \le x \le u - \sigma/\xi$. The parameters σ and ξ are called the scale and shape parameters respectively. When $\xi < 0$ the distribution has a finite end point (i.e. a maximum value, equal to $u - \sigma/\xi$) and is referred to as short tailed. When $\xi > 0$ the distribution is unbounded from above and referred to as heavy tailed or long tailed. When $\xi = 0$ the distribution has an exponential tail. The expected range of shape parameters for most environmental variables is $-0.5 < \xi < 0.5$.

The threshold is selected by fitting the distribution for a range of threshold values and selecting the threshold as the lowest value for which the shape parameter of the distribution and estimates of high quantiles converge to steady values. The method used to estimate the parameters of the GPD can have a significant influence on the accuracy of the results [17]. For the present work, the empirical Bayesian method [18] has been used. This method is computationally efficient and gives lower bias and variance in parameter and quantile estimates than commonly used methods such as maximum likelihood or probability weighted moments (see [19] for a recent comparison of estimators for the GPD).

Once the joint PDF p(a, b, k) has been estimated, the distribution of the maximum wave height in a random storm exceeding the threshold level can be calculated by integrating the short-term distribution of the maximum wave height in a storm specified by the GEV parameters a, b and k over the long-term joint distribution of a, b and k:

$$\Pr(H_{max} \le h|RS) = \int_{k=-\infty}^{k=1/2} \int_{b=0}^{b=\infty} \int_{a=u}^{a=\infty} [p(a,b,k)] \Pr(H_{max} \le h|ES(a,b,k))] \, da \, db \, dk$$
(8)

Finally, return periods are calculated as follows. The wave height that is exceeded, on average, once every m storms, is the solution of:

$$\Pr(H_{max} \le H_m | RS) = 1 - \frac{1}{m} \tag{9}$$

The wave height that is exceeded once every T years on average, is known as the T-year return value, denoted H_T , and is said to have a return period of T years. If there are on average vstorms exceeding the threshold level every year, then the T-year return value, H_T , is the value exceed every vT storms:

$$\Pr(H_{max} \le H_T | RS) = 1 - \frac{1}{\nu T}$$
(10)

The value ν is estimated as $\nu = k/\tau$ where τ is the length of the dataset in years and k is the number of storms in the dataset which exceed the threshold value used in the POT analysis.

3. THE MONTE CARLO METHOD

The Monte Carlo (MC) method is essentially very simple. The idea is that if a long time series of individual wave heights was available, then the distribution $Pr(H_{max} \le h|RS)$ could be estimated directly from the data, without the need to fit equivalent storms. In the MC approach proposed here, the required time series of individual wave heights are simulated from the time series of sea states and assumed model for the short-term distribution of wave heights, conditional on sea state.

The first step of the method is to generate a random realisation of the maximum wave height in each sea state. Next, a POT analysis is conducted on the simulated maximum wave heights and the GPD is fitted to the declustered threshold exceedances. The fitted GPD is an estimate of $Pr(H_{max} \le h|RS)$ for this particular realisation. To capture the random variability in the simulated maximum wave heights, this process is repeated n times and the average of the estimated GPD parameters is taken over the n trials to obtain a stable estimate of $Pr(H_{max} \le h|RS)$. Return periods of individual wave heights can then be calculated in the same way as for the ES model.

Before going into details, it is important to note that part of the POT analysis involves selecting a threshold. It is not sensible to select the threshold individually for each trial, so the threshold must be pre-selected somehow. This could be done by selecting a threshold for a number of random samples and taking the mean. However, it is reasonable to expect that an appropriate threshold could be chosen by selecting the threshold based on a deterministic set of maxima for each sea state, such as the mean, mode or median value of $Pr(H_{max} \le h|\sigma)$. If the short-term distribution of individual wave heights is a Weibull distribution, then asymptotic arguments can be used to give a closed form expression for the mean or mode. However, the median value is always simple to calculate, regardless of the form of the shortterm distribution. From (1), the median value of the distribution of the maximum wave height in a sea state, $H_{0.5}$, is simply:

$$H_{0.5} = F_{\sigma}^{-1} \left(0.5^{\frac{1}{N}} \right) \tag{11}$$

The median value of $Pr(H_{max} \le h|\sigma)$ is therefore used to select the threshold for the POT analysis. Once (11) has been solved for each sea state, the data are declustered and the threshold is selected in same way as described in Section 2.

To simulate a random value for the maximum wave height in a sea state it is not necessary to simulate all waves in the sea state. Instead, a single random value can be generated from the distribution of the maximum wave height $F_{\sigma}(h)^N$. Suppose that there are M sea states in the metocean dataset. A series of Mindependent uniformly distributed variables $P_i \in [0,1], i =$ 1, ..., M are generated. The random maximum in each sea state is then the solution of

$$H_{max,i} = F_{\sigma,i}^{-1} \left(P_i^{\frac{1}{N_i}} \right) \tag{12}$$

Many models for the short-term distribution assume that individual wave or crest heights follow a Weibull distribution:

$$F_{\sigma}(h) = 1 - \exp\left(-\left(\frac{h}{\alpha H_s}\right)^{\beta}\right)$$
(13)

where the parameter α and β are dependent on sea state. In this case (12) can be written in closed form:

$$H_{max,i} = \alpha_i H_{s,i} \left(-\log\left(1 - P_i^{\frac{1}{N_i}}\right) \right)^{\frac{1}{\beta_i}}$$
(14)

By generating vectors of α , β , N, and P for each sea state, the simulation of a random realisation of the maximum, takes a fraction of a second for a 30-year hourly time series. Even if the assumed form of the short-term distribution does not allow a closed for solution for the inverse, interpolation of the distribution function $F_{\sigma}(h)$ to the value corresponding to $P^{1/N}$ typically gives an efficient means for the simulation of the maximum wave height in each sea state.

Once the random series of sea state maxima has been generated, the data are declustered to identify independent storm peak wave heights. Independent storm peaks are defined in the same way as for the ES method, described in Section 3, with a requirement that peaks must be separated by a minimum of 5 days and the minimum value between two adjacent peaks must be less than 50% of the lower of the two peaks. Once the data is declustered the GPD is fitted using the pre-selected threshold and the process is repeated n times. In the examples considered in the following section it was found that taking the mean of the estimated GPD parameters over 1000 random trials is sufficient to establish a stable estimate of return periods.

The steps required to calculate $Pr(H_{max} \le h|RS)$ for the MC and ES methods are summarised in Fig. 1. The MC method is a significantly simpler process than the ES method, requiring only one fitting stage, compared to three. The individual steps in the MC method are also much simpler to implement in a computer program and require less input from the user in selecting models for the data.



FIGURE 1. Flow charts of steps required to calculate $Pr(H_{max} \le h|RS)$ for the Equivalent Storm and Monte Carlo methods.

4. APPLICATION TO MEASURED DATA

The MC approach is illustrated here by applying the method to a long time series of wave buoy measurements. The

data used here comes from the US NDBC buoy number 46014, located off the coast of Northern California in a water depth of 256m. The dataset consists of 35 years of hourly records of wave spectra over the period April 1981 – December 2016. In this example, the long-term distribution of individual crest heights will be calculated, assuming the short-term distribution follows the Forristall second-order model for directionally spread seas [20]. The Forristall model assumes a Weibull distribution of crest heights (equation (13)) with the parameters α and β defined in terms of the significant steepness, s_m , and Ursell number, U_{rs} , defined as

$$s_m = \frac{2\pi H_s}{gT_m^2}, \qquad U_{rs} = \frac{H_s}{k_m^2 d^3}$$
 (15)

where k_m is the finite depth wave number corresponding to T_m and d is the water depth. The distribution parameters are given by

$$\alpha = 0.3536 + 0.2568s_m + 0.0800U_{rs}$$

$$\beta = 2 - 1.7912s_m - 0.5302U_{rs} + 0.2824U_{rs}^2$$
(16)

4.1 Application of the equivalent storm method

The application of the ES method to this dataset is described in detail in [12], and will be summarised here for completeness. The time series of sea states was divided into storms using the criteria defined in Section 2, which resulted in 1036 separate storms, and the GEV was fitted to the distribution of the maximum crest height in each storm. The relationship between the fitted GEV parameters is shown in Fig. 2. It is evident that *a* and *b* are strongly linearly correlated and that the shape parameter *k* is uncorrelated with either *a* or the residuals $b - b_{fit}$, where $b_{fit} = \gamma + \delta a$ is the regression line shown on the top plot. Figure 3 shows the distributions of *k* and the residuals $b - b_{fit}$. Both distributions are well modelled by a Student's tdistribution.

The GPD was fitted to the GEV location parameter, a, for threshold levels between 3 and 9m. For each threshold level, the dataset was resampled using a bootstrap technique to give confidence bounds on the parameter estimates. The variation of the shape parameter ξ and the estimated 100-year return value of a with the threshold level is shown in Fig. 4. It is evident that for threshold levels above 5m, the shape parameter and return values are approximately stable (within the confidence bounds) and that for threshold above 7m there is a very high uncertainty in the shape parameter, due to the small number of threshold exceedances at this level. Based on this analysis, a threshold level of 5m was selected. Figure 5 shows the fit of the GPD to the location parameter, a, for this threshold level. It is evident that the GPD is a good fit for the data.

Finally, it is worth noting that due to the narrow range of the distributions of k and the residuals $b - b_{fit}$, the triple integral in (8) can be replaced by a single integral over p(a), using the mean value of k and the mean value of b conditional on a (i.e. the linear model shown in Fig. 2). In [12] it was shown that the

reduction from the triple integral (8) to the single integral (17) has no discernible effect on the estimated return values.



FIGURE 2. Relations between fitted GEV parameters.

4.2 Application of the Monte Carlo method

For the MC method, the only step requiring user input is the selection of the threshold level for fitting the GPD. Before discussing this, it is instructive to consider the variables that the GPD is fitted to in each method. For the MC method, the GPD threshold is selected by fitting to the maximum value of $H_{0.5}$ in each storm (where $H_{0.5}$ is the median value of the distribution of the maximum crest height in each sea state, defined in equation (11)). For the ES method, the GPD is fitted to the GEV location parameter, *a*. Figure 6 shows the relationship between the GEV location parameter *a* and the most probable maximum crest height in the storm, H_{mp} (i.e. the mode of the distribution given by equation (2)). The two variables are almost identical. The reason for this is that when the GEV shape parameter *k* is equal to zero, the mode of the distribution occurs at h = a. As the GEV distributions fitted to equation (2) have k very close to zero, the mode occurs very close to h = a, resulting in the relationship shown in Fig. 6.



FIGURE 3. Histograms and fitted Student's t-distributions for GEV shape parameter k and residuals of b.







FIGURE 5. Fit of the GPD to GEV location parameter *a* for a threshold of 5m.



FIGURE 6. Relationship between GEV location parameter a and H_{mp} . Red dashed line is linear fit.



FIGURE 7. Relationship between maximum value of $H_{0.5}$ in a storm and most probable maximum crest height H_{mp} . Black dashed line is 1:1 relation.

Figure 7 shows the relationship between H_{mp} and the maximum value of $H_{0.5}$ in each storm. It is evident that H_{mp} is slightly higher than max $(H_{0.5})$, but that the two variables are strongly correlated. Therefore, it would be expected that a suitable threshold for the MC method will be very close to the threshold level selected for the ES method and the quality of the fit of the GPD in each method will be similar. For the current example, the threshold for the MC analysis has been set at the same value as used for the ES analysis, at a level of 5m. The quality of the fit of the GPD to the values of max $(H_{0.5})$ at this threshold level is very similar to the fit of the GPD to H_{mp} , shown in Fig 5 and is therefore not shown here.

4.3. Results and discussion

Figure 8 shows the return values of maximum crest height at return periods between 1 and 1000 years, calculated from each method, using a threshold level of 5m. The estimates from the two methods are in very close agreement, with the maximum differences less than 0.12m throughout the range. Some small differences between the methods are to be expected, as the POT analysis is conducted on different variables for the two methods and there is uncertainty related to the subjective choice of threshold level (see Fig. 4). Although the same threshold level has been used for both methods, the GPD is fitted to different variables in each approach, so there will be some differences in the results.

To assess the uncertainty associated with the choice of threshold level, the two methods were applied for thresholds between 5 and 6m, at intervals of 0.1m. The maximum and minimum return values of maximum crest height for this range of thresholds is shown in Fig. 9 for each method. The uncertainty related to threshold level is similar for each method, with both methods having a range of 0.3m at the 100-year level and 0.5m at the 1000-year level. The MC method gives slightly higher results on average at the 100-year level, but the range of values from different threshold overlaps for the two methods. It is therefore concluded that the two methods agree within the uncertainty due to threshold level.

5. CONCLUSIONS

A new method for calculating return periods of individual wave and crest heights has been introduced. The new Monte Carlo (MC) method is significantly simpler than the equivalent storm (ES) method and gives almost identical results. The Monte Carlo method allows the distribution of the maximum wave or crest height in a random storm, $Pr(H_{max} \le h|RS)$, to be estimated directly from the data, without the need for calculating distributions of the maximum height in the storm, fitting equivalent storms or estimating joint distributions.

The steps required to calculate $Pr(H_{max} \le h|RS)$ for each method are summarised in Fig. 1. The MC method contains only one fitting stage, rather than the three required in the ES method. It is therefore likely to be more generally applicable and robust, as fewer assumptions are required. In terms of computational effort, the MC method is much simpler to implement and has

similar computational times. In the ES method, the computational effort is in the fitting of the equivalent storms, whereas in the MC method the computational effort is in the number of iterations required. For the 35 year dataset considered in this paper, both methods took around 1 minute to compute on a Windows 10 laptop with an Intel Core i7-8550U processor.

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FIGURE 8. Comparison of return values of maximum crest heights from Equivalent Storm and Monte Carlo methods for a threshold level of 5m.



FIGURE 9. Minimum and maximum return values of maximum crest heights for threshold levels between 5 and 6m from Equivalent Storm and Monte Carlo methods.

REFERENCES

- Battjes JA. "Long-term wave height distribution at seven stations around the British Isles". National Institute of Oceanography, Internal Report No. A44, 31pp. (1970)
- [2] Tucker MJ. "Improved 'Battjes' method for predicting the probability of extreme waves". *Applied Ocean Research*, 11, 4 (1989): 212–218.
- [3] Krogstad HE. "Height and period distributions of extreme waves". *Applied Ocean Research*, 7 (1985): 158–165.
- [4] Tucker MJ and Pitt EG. *Waves In Ocean Engineering*. Elsevier, Amsterdam (2001).
- [5] Tromans PS and Vanderschuren L. "Response based design conditions in the North Sea: Application of a new method". *Proc. Offshore Tech. Conf.*, OTC 7683, 1995.
- [6] Boccotti P. *Wave Mechanics for Ocean Engineering*, Elsevier, New York (2000).
- [7] Arena F and Pavone D. "The return period of non-linear high wave crests". *Journal of Geophysical Research* 111(2006): C08004.
- [8] Fedele F and Arena F. "Long-term statistics and extreme waves of sea storms". *Journal of Physical Oceanography*, 40, 5, (2010): 1106–1117.
- [9] Arena F, Malara G, Romolo A. "On long-term statistics of high waves via the equivalent power storm model". *Probabilistic Engineering Mechanics*, 38 (2014), 103-110.
- [10] Martín-Soldevilla MJ, Martín-Hidalgo M, Negro V, López-Gutiérrez JS, Aberturas P. "Improvement of theoretical storm characterization for different climate conditions". *Coastal Engineering*, 96 (2015), 71–80.

- [11] Laface V and Arena F. "A new equivalent exponential storm model for long-term statistics of ocean waves." *Coastal Engineering*, 116 (2016): 133-151.
- [12] Mackay EBL. "A Generalised Equivalent Storm Model for Long-Term Statistics of Ocean Waves". Submitted to *Coastal Engineering*.
- [13] Forristall GZ, 2008. "How should we combine long and short term wave height distributions?" Proc 27th Int. Conf. Offshore Mech. Arctic Eng. (OMAE2008-58012). Estoril, Portugal, 15–20 June 2008.
- [14] "Environmental conditions and environmental loads". DNV GL Recommended Practice RP C205, August 2017.
- [15] Davis RA, Mikosch T. "The extremogram: a correlogram for extreme events". Bernoulli 15, (2009) 977–1009.
- [16] Lagarias JC, Reeds JA, Wright MH, Wright PE. "Convergence Properties of the Nelder-Mead Simplex Method in Low Dimensions." SIAM Journal of Optimization. 9, 1 (1998): 112–147.
- [17] Mackay EBL, Challenor PG, Bahaj AS. "A comparison of estimators for the generalised Pareto distribution". Ocean Engineering, 38, 11–12, (2011), 1338-1346.
- [18] Zhang J. "Improving on estimation for the generalized pareto distribution". Technometrics, 52 (2010), 335–339.
- [19] Kang S, Song J. "Parameter and quantile estimation for the generalized Pareto distribution in peaks over threshold framework". Journal of the Korean Statistical Society 46 (2017) 487–501.
- [20] Forristall GZ. "Wave crest distributions: Observations and second order theory". *Journal of Physical Oceanography*, 30, 8, (2000): 1931–1943.